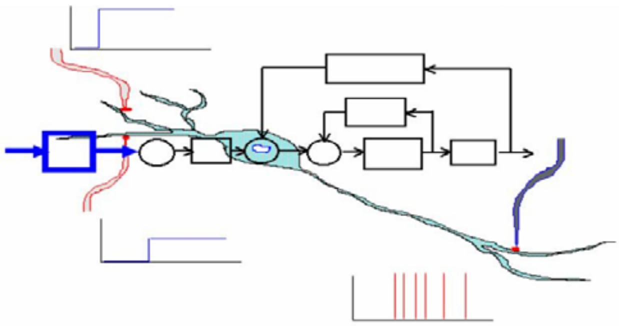
Summer School on Intelligent Systems
University of Cyprus
July 2-6, 2007



Single neuron models: A comparative review

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Single neuron models: A comparative review

Introduction

Biological and artificial neural networks

Biological neuron

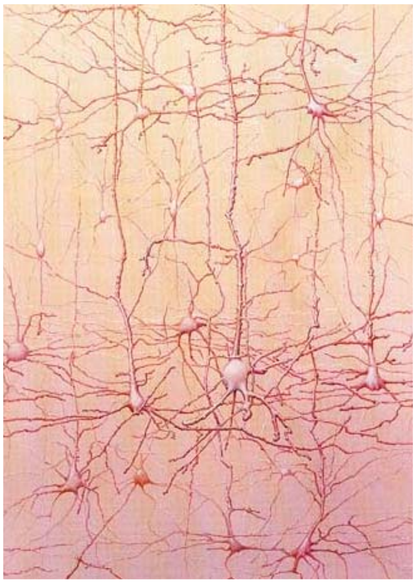
Biological neuron models

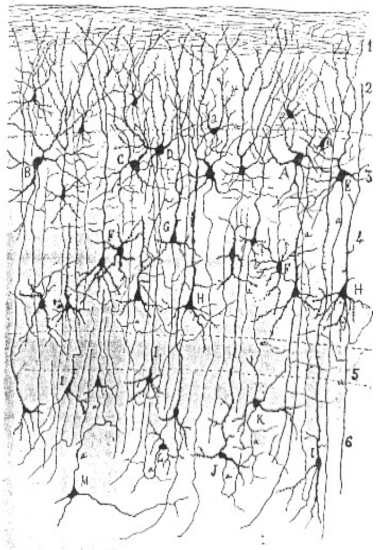
Artificial neuron

Artificial neuron models

Comparisons

Introduction





BRAIN – NERVOUS SYSTEM

The biological neural networked systems (BNN) are **highly interconnected adaptive structures** composed of a vast number ($\sim 10^{12}$) of **non-linear processing elements (PE)**, or units, or biological neurons (BN).

It is estimated that the total length of the brain connections are about 10^9 meters, which is about 25 times the perimeter of the earth!

The PE **operate simultaneously** (parallel processing), directly or indirectly **influencing one another**, working **cooperatively** in a **concerted manner**.

Because of parallelism, the system exhibits characteristics of **robustness**, **fault-tolerance** and **fuzzy value processing**.

BRAIN – NERVOUS SYSTEM

The BNN have important capabilities such as the capacity for **learning**, **memorizing** and **information retrieval**.

The biological neural networked systems can easily execute tasks such as **recognition**, **generalization**, **forecasting** and many other **higher cognitive**, **perceptive**, **emotional**, **behavioral** and generally **sentimental tasks** (calculations, language, love, consciousness, …).

These **emerge naturally** in manners that are largely unknown.

i.e complex behavioral systems and patterns arise out of simple interactions of a multiplicity of relatively simple units.

ARTIFICIAL – NERVOUS SYSTEM

The ANNs are structures that aim to mimic the operational characteristics of natural (biological) neural networks, and possibly (or hopefully) to improve on these.

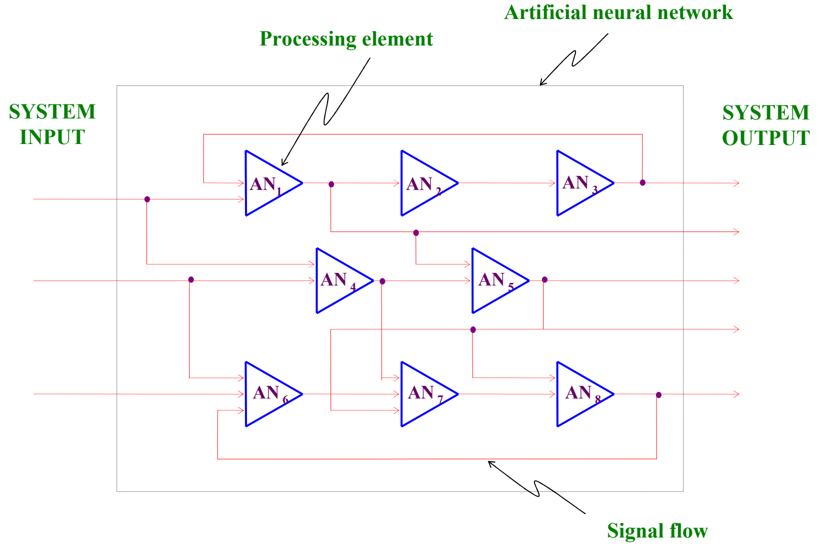
They are composed of many **artificial neurons** (AN) connected in a system, usually of an organized pattern, in which there is direct or indirect communication and interaction among all its members.

There is usually provision for information **input** and for the desired **output**.

Groups of neurons may be organized into **layers** or **slabs**, or any other desired formalism.

An Artificial Neural Network

An **emerging global behavior** may appear through the use of simple local learning, and possibly evolving, rules.



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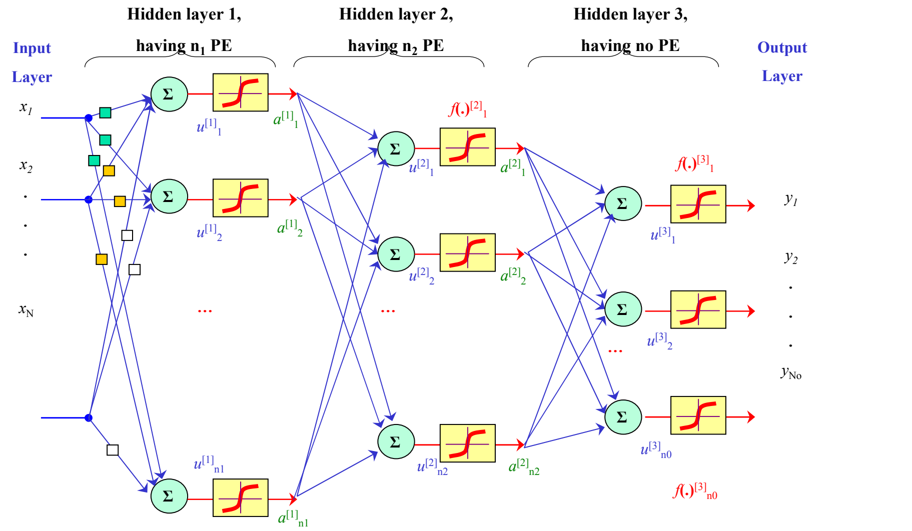
TYPICAL GRAPHICAL FORMALISM FOR ANN REPRESENTATION



$$y_l^{[out]} = y_l^{[3]} = f_l^{[3]}(u_l^{[3]}) = f_l^{[3]}\left(\sum_{k=1}^{n_2} a_k^{[2]} w_{kl}^{[3]}\right) = f_l^{[3]}\left(\sum_{k=1}^{n_2} f_k^{[2]}(u_k^{[2]})\, w_{kl}^{[3]}\right) = \dots$$

$$\dots = f_l^{[3]}\left(\sum_{k=1}^{n_2} f_k^{[2]}\left(\sum_{j=1}^{n_1} f_j^{[1]}(u_j^{[1]})\, w_{jk}^{[2]}\right) w_{kl}^{[3]}\right) = f_l^{[3]}\left(\sum_{k=1}^{n_2} f_k^{[2]}\left(\sum_{j=1}^{n_1} f_j^{[1]}\left(\sum_{i=1}^{N} x_i w_{ij}^{[1]}\right) w_{jk}^{[2]}\right) w_{kl}^{[3]}\right) \qquad l = 1, \dots, N_o$$

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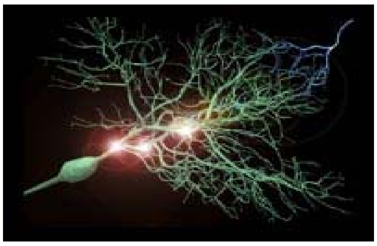
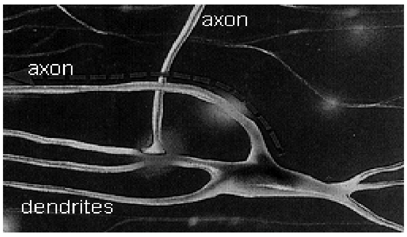
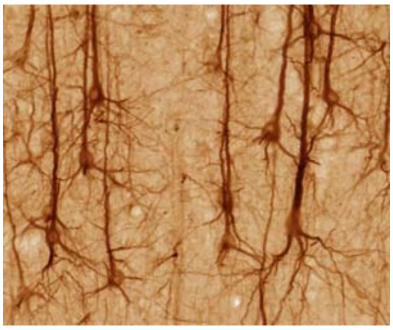
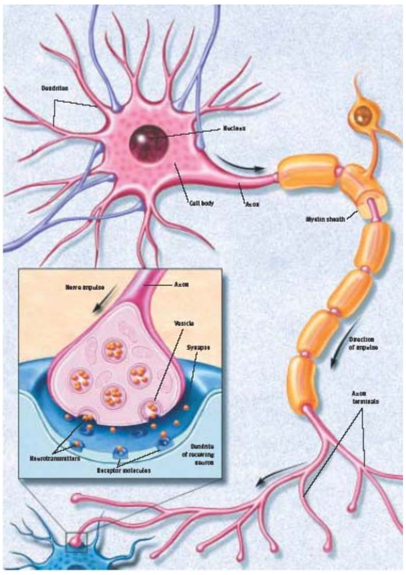
BIOLOGICAL SINGLE NEURONS









BIOLOGICAL NEURONS (BN)

They are the **basic building blocks** of a biological neural system (brain).

They are **relatively slow** compared to modern silicon gates *(slower by about 5 to 10 orders of magnitude)*.

The information transmission is largely done through **electrochemical** processes.

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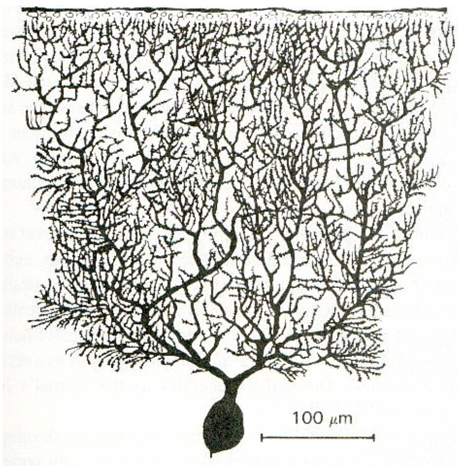
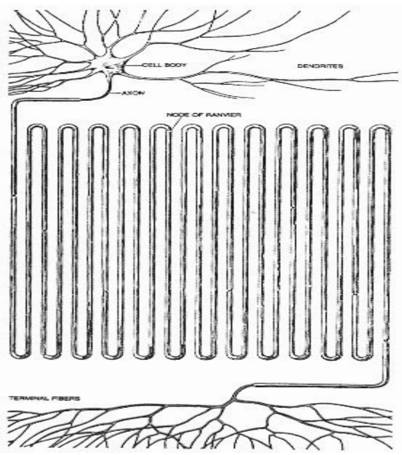
Some basic biological neurons



Purkunje
cell at the cerebellum



Large axon BN

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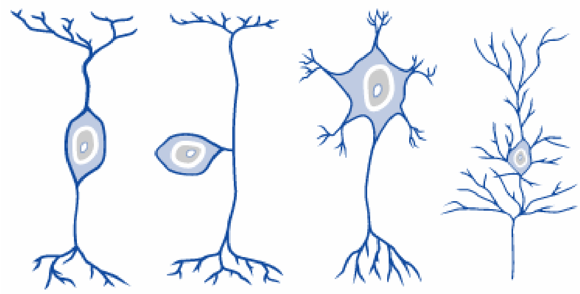
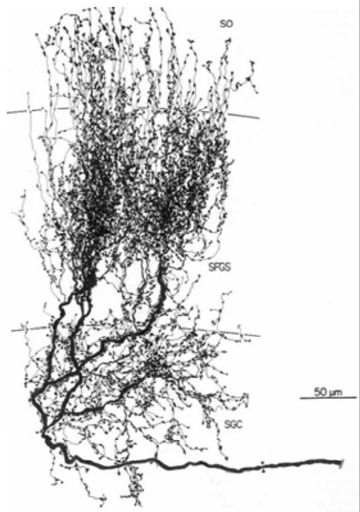
There are about 1000 different kinds of neurons in the brain !





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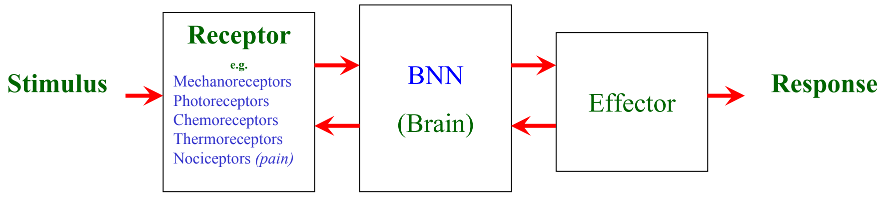
Information transmission



e.g.

Light in eye,

Pressure in finger

e.g

Closure of eye lids,

Pain

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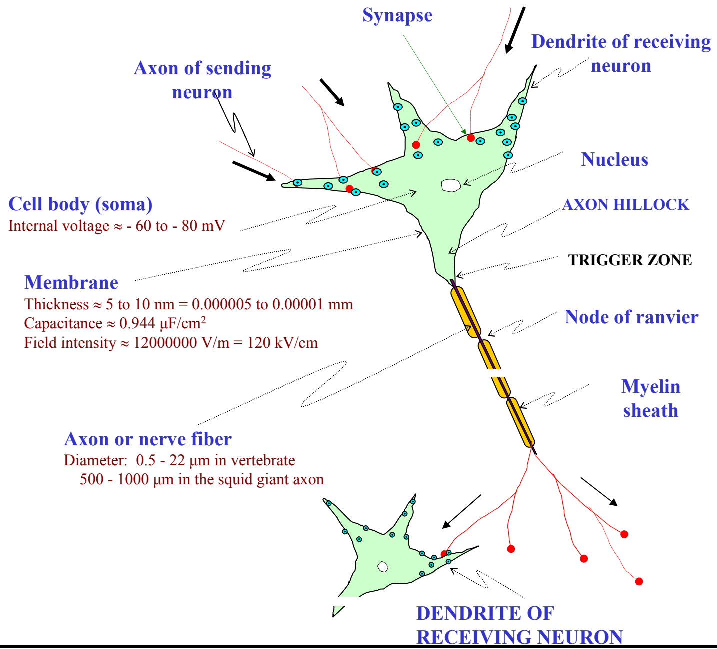
More detailed (but still encyclopedic) **look on the biological neuron**



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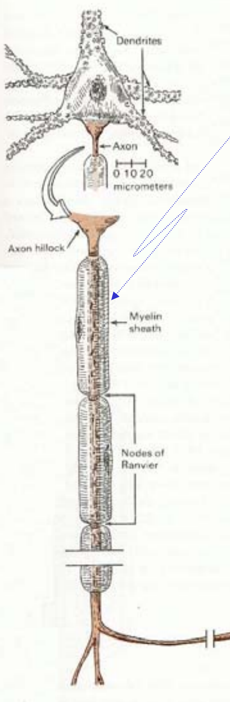
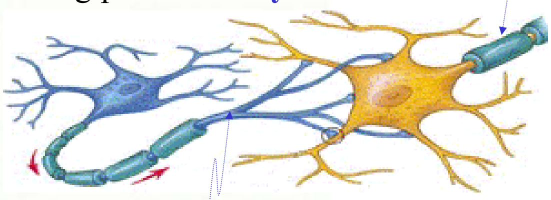


The **axons,** mainly of the peripheral nervous system, are enclosed by other neurons (**Schwann cells** or neurolemmocytes) to create an insulating/protective **myelin sheath**.



The **length of an axon** can be large (about 1m in some human motor neurons) or very small as for instance the amacrine neuron cells in the retina.

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Synapse

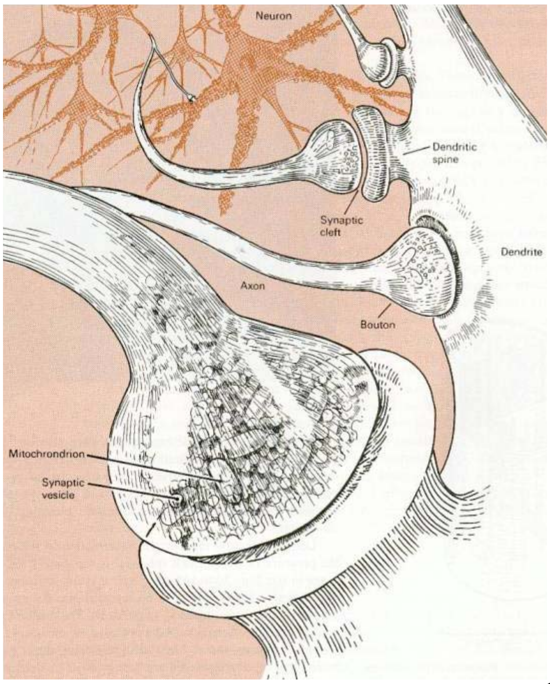
Diameter ≈ 1 μm = 0.001mm

Gap ≈20 to 40 nm
= 0.00002 to 0.00004 mm

Diameter/Gap ratio ≈ 100

Delay in the transmission of the pre-synaptic potential to a post-synaptic potential ≈ 0.3 to 1.0 ms

Velocity ≈ 0.2 cm/minute



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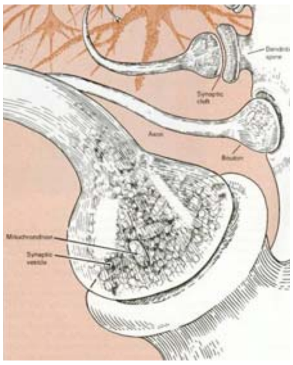
The **medium** for the signal transmission is basically of electrochemical nature.

There exists a great variety of synapses.

Even the **position** of a synapse may result in significant difference in signal transmission.



The transmission of coded information at the synapses is primarily done with chemical substances known as **neurotransmitters**
(e.g. acetylcholine, norepinephrine, dopamine, 5-hydroxytryptamine serotonin, aminobutyric acid GABA).

These phenomena are mainly occurring in the mammal nervous systems.

There also exist electrical synapses, but these occur mainly in lower animals.

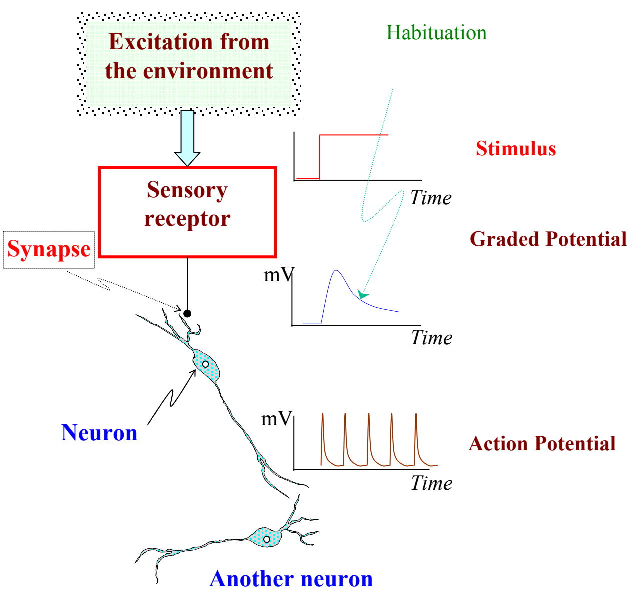
The systematic use of a synapse is believed to improve its efficacy.

→ **learning, memory**

Hebb's Rule

INFORMATION TRANSMISSION

(Generation of the Action Potential)



The receptive sensors convert changes in the environment (light, pressure, chemical constituency, ...) in Graded Potential (GP) (postsynaptic dendritic potential, transducer potential, or input potential).

The graded potential is gradually weakened (decaying) as it is propagated towards the soma.

Many graded potentials from different dendrites are **accumulated** to create a bigger or smaller one, which, if it reaches a threshold of about –40mV **at the trigger zone**, it becomes an **Action Potential** (AP) – (a train of pulsed voltages, nerve impulses or spikes)

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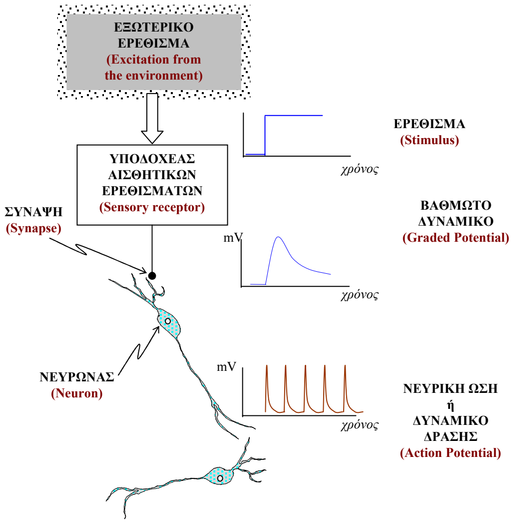
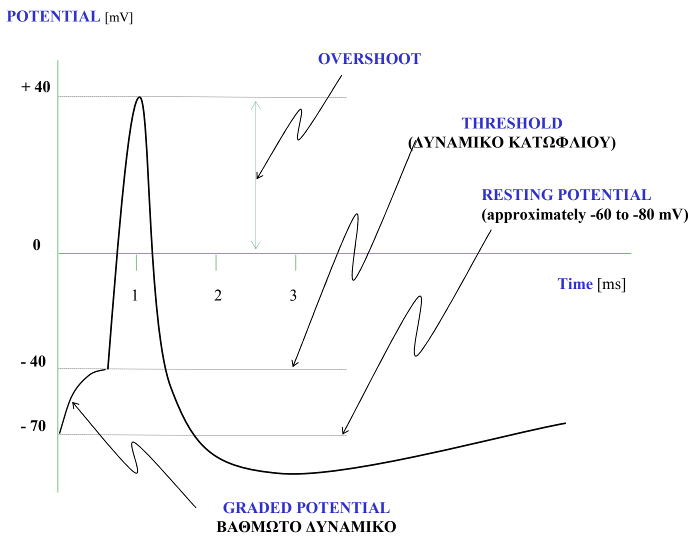
So, the system may simply be thought as a voltage to frequency converter, in a similar way that is used in some communication systems.

Obviously, the synapses that are closer to the axon they have a higher probability in contributing to the generation of the action potential.



Action potential (or spike) (AP)



Some characteristics of the Action potential

Wavelength ≈12 cm

Amplitude ≈110 mV above the **resting potential** (RP)

The **amplitude** of the AP does not diminish as it is propagated along the axon.

Speed of propagation ≈ 0.5 - 120 m/s
(1.8 - 432 Km/hour)

It depends mainly on the axon diameter and on the presence or not of the myelin sheath.

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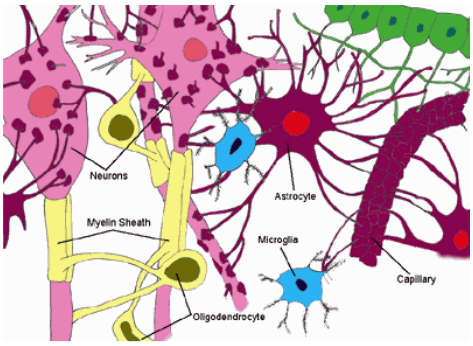
The information transmission in synapses is done in parallel.

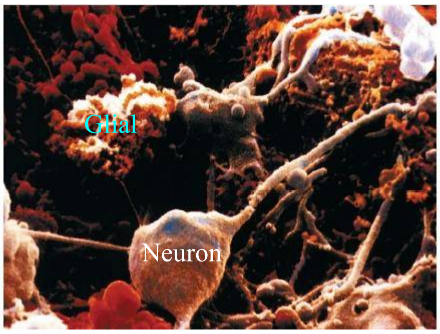
Because of this, the frequency of changes is about 10^{16} per second.

It was believed that synapses that are nearer an axon contribute more towards a generation of an Action Potential.
Modern computer simulations though, indicate that this is not always true, mainly due to the non-linearity of processing.

Taking into consideration the fact that at each synapse many different kinds of neurotransmitters are also transferred, **the information transmission is impressive**.

GLIAL CELLS (or Neuroglia or Glia)





These are **non-neuronal cells** *(do not generate AP, do not have synapses)* that **provide support** and **nutrition**, **maintain homeostasis**, **form myelin (Schwann cells)**, and **participate in signal transmission** in the nervous system.

They compose most of the n mass (~ 90%).

They also help for the **support and guidance of embryonic neurons**.

There are **indications that they communicate with the neuron cells and among themselves**
(Stephen Smith, Yale U., 1993).

There are **five different types** of glial cells:

Astrocytes:	Provide physical and nutritional support. Digest part of dead neurons. Regulate the extracelullar fluid.
Microglia:	Digest part of dead neurons.
Oligodendroglia:	Provides insulation for neurons (myelin).
Satellate cells:	Provide physical support.
Schwann cells:	Provide insulation for neurons (myelin).

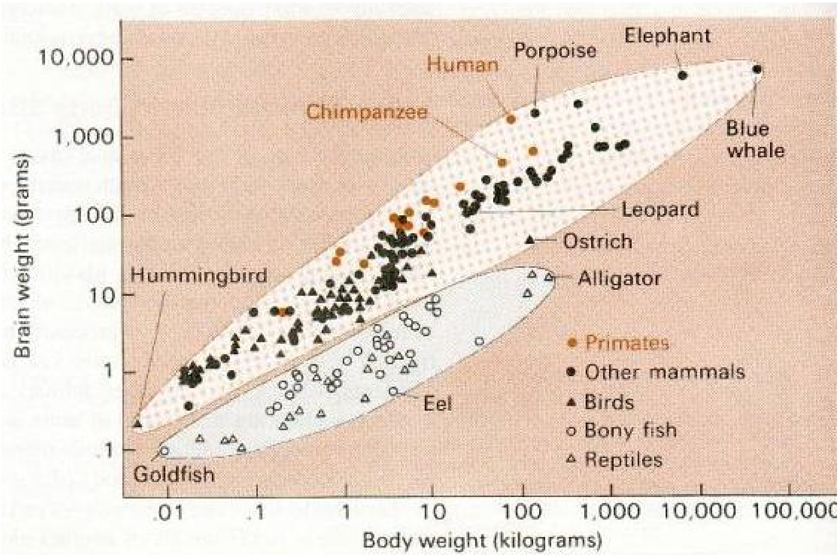
Comparisons

There are about $10^{10} – 10^{12}$ neurons in the brain

and about $10^{13} – 10^{16}$ synapses.

Organism	**leech**	**Worm**	**Fly**	**Cockroach**	**Bee**	**Man**
Number of Synapses	**$>10^4$**	**$>10^5$**	**$\sim 10^9$**	**$<10^{11}$**	**$>10^{11}$**	**$\sim 10^{14}$**

Comparisons



Models of Biological Neurons

In the following section, a list of **the most important mathematical models of single biological neuron** cells will be presented.

The most **important features** (functional and structural) will be identified, mainly to be appraised as possible valuable characteristics in building useful artificial neuron models.

It is not the objective to get involved in the intricate chemical and physiological processes, that may be of interest to a neurologist.

The following information on the mathematical models of BN is rather intended to help gain an understanding of the various structures of the models.

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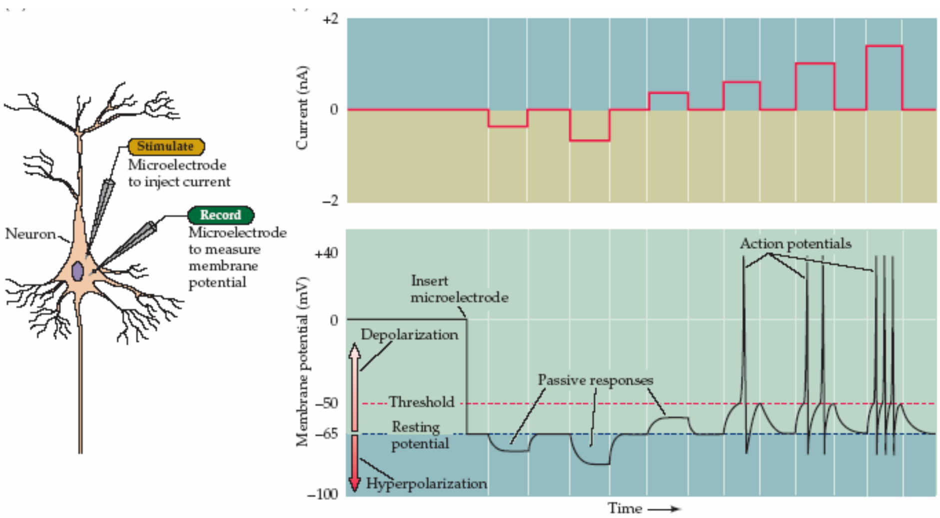
Generation of the Action Potential



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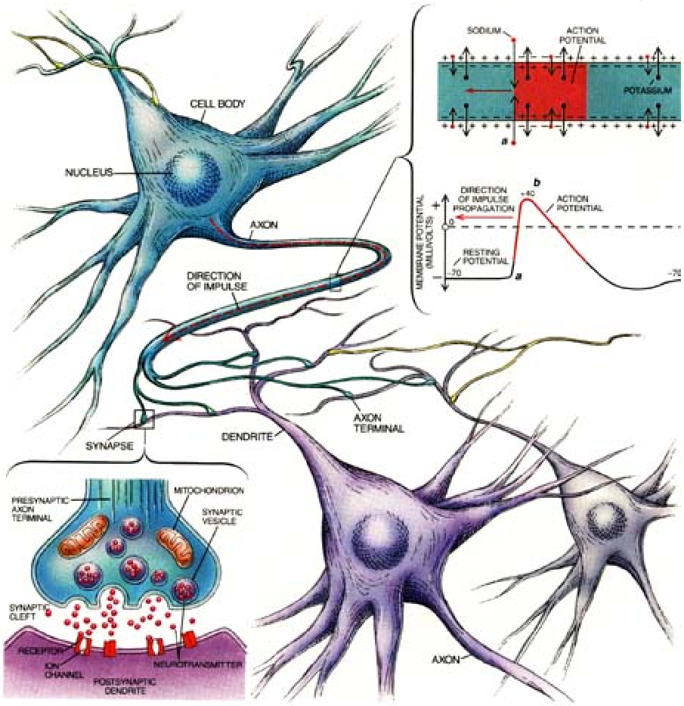
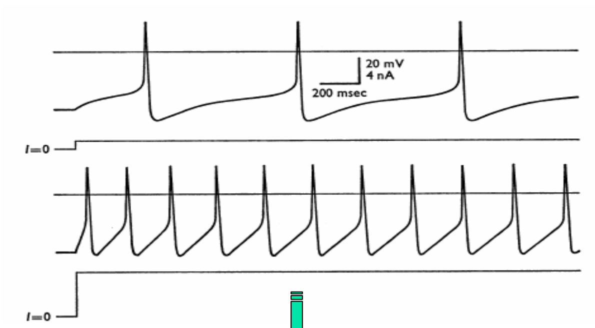
Generation of the Action Potential

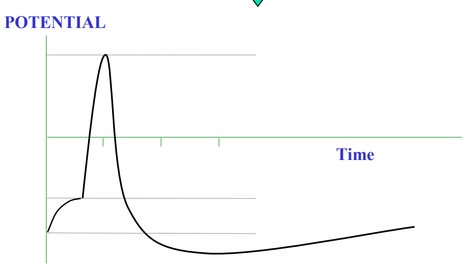


Modeling the Action Potential

SPIKE TRAINS AT DIFFERENT FREQUENCIES



AN ACTION POTENTIAL



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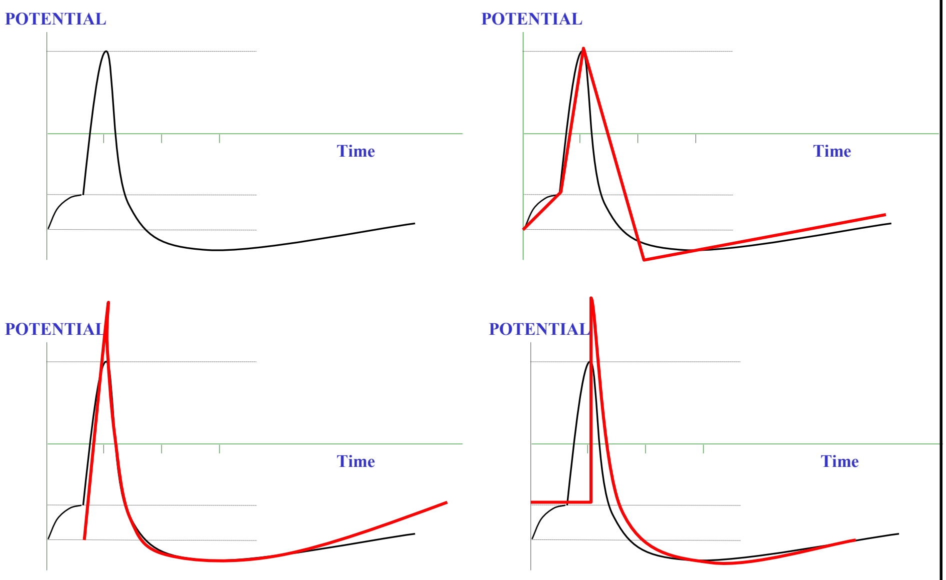
Modeling the Action Potential



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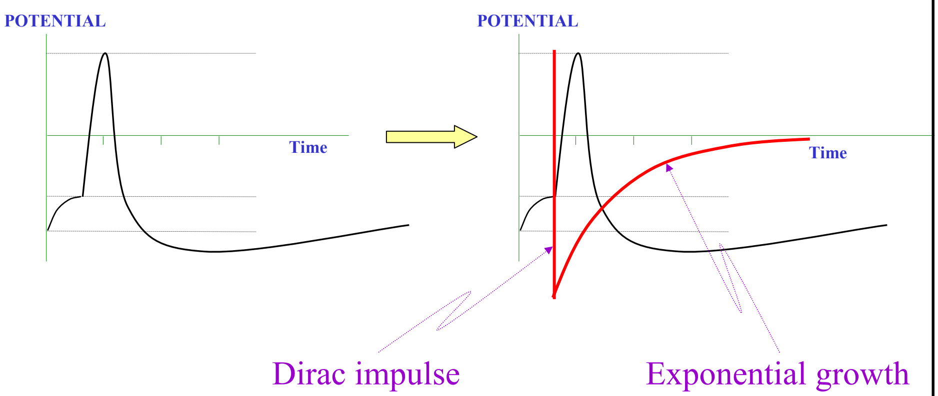
Modeling the Action Potential

A simple model of the AP that is commonly used



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Models of Biological Neurons

1. Conductance-based models of BN

These are based on modeling the cell membrane by an equivalent electric circuit.

Typical conductance-based models:

Hodgkin-Huxley, (1952)

Fitzhugh-Nagumo, (1961)

Connor-Stevens, (1971)

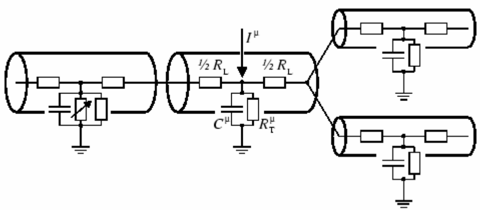
Morris-Lecar, (1981)

2. Compartmental-based models of BN

Because the structure of real bio-neurons is highly complex, it is difficult to model the important parameters and characteristics through the use of analytic conductance-based models.

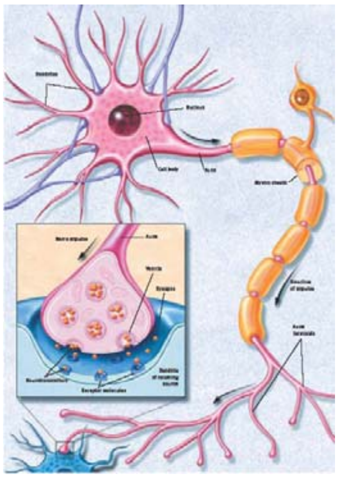
A more detailed description of neuron dynamics can be obtained through the use of compartmental-based models, which essentially break down a complex system/structure into granules of interacting but simpler building blocks.

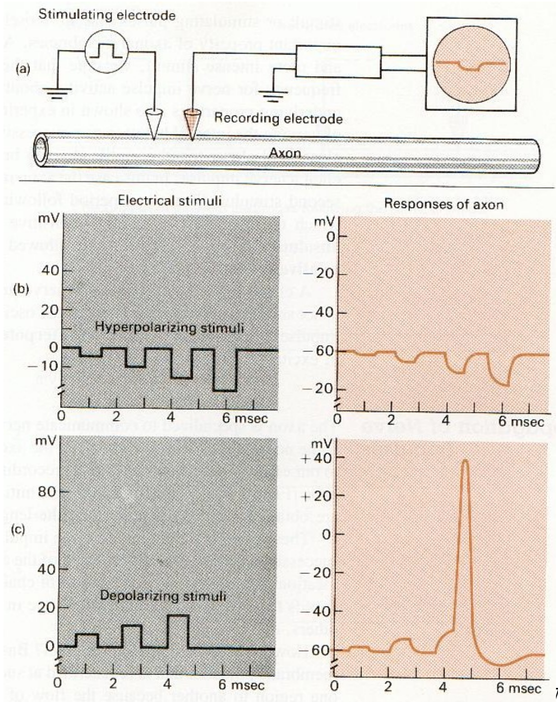
A compartmental model of a dendritic section



Mathematical models of BN

1. Conductance-based models:

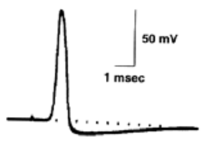




The Hodgkin – Huxley (HH) model, (1952)

(1963 Nobel prize in physiology and medicine)

The model was developed by using data obtained from experiments done on the **giant axon** of the **squid** (diameter ≈ 0.5 mm).

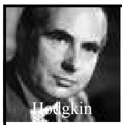


The 4-dimensional mathematical/quantitative HH model is given below:

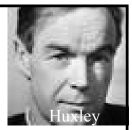
$$C_m \frac{dV}{dt} = I_m - \underbrace{g_{\mathrm{Na}} m^3 h (V - V_{\mathrm{Na}})}_{I_{Na}} - \underbrace{g_{\mathrm{K}} n^4 (V - V_{\mathrm{K}})}_{I_K} - \underbrace{g_{\mathrm{L}} (V - V_{\mathrm{L}})}_{I_{Leakage}}$$

$$\frac{dm}{dt} = \alpha_m (1 - m) - \beta_m m \qquad \frac{dn}{dt} = \alpha_n (1 - n) - \beta_n m \qquad \frac{dh}{dt} = \alpha_h (1 - h) - \beta_h m$$

Hodgkin A., Huxley A. (1952). A quantitative description of membrane current and its application to conduction and excitation in nerve. **J. Physiol. 117**:500–544.



The Hodgkin – Huxley model



where,

I_m = Membrane current *(the excitation)*
C_m = Membrane capacitance
V_{rest} = – 60 mV
V_{na} = 50mV
V_K = – 77 mV

g_{Na} = 120 mmho/cm^2
g_K = 36 mmho/cm^2
g_L = 0.3 mmho/cm^2

$E = V - V_{rest}$

$$\alpha_m = \frac{0.1(25-E)}{\mathbf{e}^{\frac{25-E}{10}} - 1} \qquad \alpha_n = \frac{0.01(10-E)}{\mathbf{e}^{\frac{10-E}{10}} - 1} \qquad \alpha_k = 0.07\mathbf{e}^{\frac{-E}{10}}$$

$$\beta_m = 4\mathbf{e}^{\frac{-E}{18}} \qquad \beta_n = 0.125\mathbf{e}^{\frac{-E}{80}} \qquad \beta_k = \frac{1}{\mathbf{1} + \mathbf{e}^{\frac{30-E}{10}}}$$

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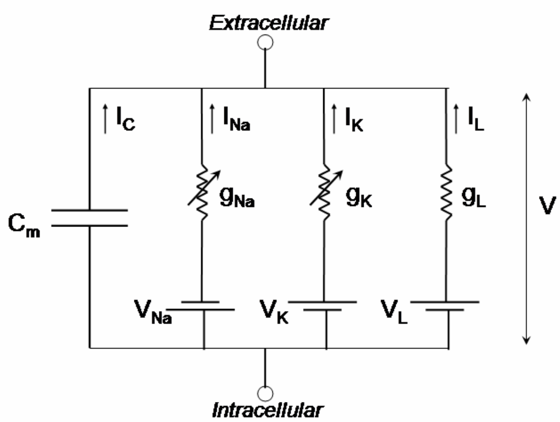
The HH model is basically a conductance based model.

An equivalent electric circuit is shown below.



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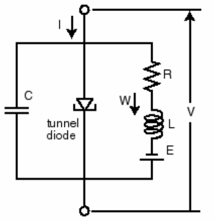
The FitzHugh-Nagumo model, (1961)

This is a **two-variable** (2D) **neuron model**.
It is a simpler form of the Hodgkin – Huxley model.

It was originally suggested by FitzHugh (1961), who called it "Bonhoeffer-van der Pol model".

Nagumo et al. proposed an equivalent electric circuit (1962).

$$\frac{du}{dt} = u - \frac{u^3}{3} - z + x$$



$$\frac{dz}{dt} = -k_1 z - k_2 u - k_3 \quad = 0.080z + 0.056u + 0.064$$

FitzHugh R. (1961). Impulses and physiological states in theoretical models of nerve membrane. **Biophysical J. 1**:445-466.

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The Connor-Stevens model, (1971)

(Connor and Stevens, 1971; Connor et al. 1977)

The membrane current in this model is given by:

$$I_m = g_{\mathrm{L}}(V - V_{\mathrm{L}}) + g_{\mathrm{Na}} m^3 h (V - V_{\mathrm{Na}}) + g_{\mathrm{K}} n^4 (V - V_{\mathrm{K}}) + g_{\mathrm{A}} a^3 b (V - V_{\mathrm{A}})$$

which, basically is the HH model having an additional K-conductance, called the A-current.

Connor J., Stevens C. (1971). Inward and delayed outward membrane currents in isolated neural somata under voltage clamp. **J. Physiol.** Feb. **213(1)**:1–19**.**

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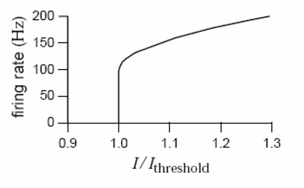
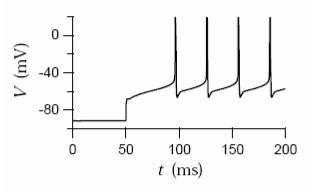
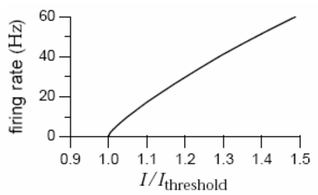
The Connor-Stevens model

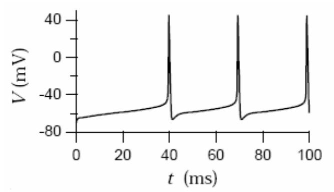
The figure below shows a typical response of the **Hodgkin-Huxley** model.





The figure below shows a typical response of the **Connor-Stevens** model.





The Morris-Lecar model, (1981)

Like the FitzHugh-Nagumo model, this is also a two-dimensional model.

They were originally formulated to describe electrical activity in barnacle muscle fiber.

The general mathematical form is given by:

$$C_m \frac{dV}{dt} = I_m - g_{\mathrm{Ca}} m (V - V_{\mathrm{Ca}}) - g_{\mathrm{K}} n (V - V_{\mathrm{K}}) - g_{\mathrm{L}} (V - V_{\mathrm{L}})$$

$$\frac{dn}{dt} = \alpha_n (1 - n)$$

Morris C., Lecar H. (1981). Voltage oscillations in the barnacle giant muscle fiber. **Biophys. J., 35**:193-213.

The Morris-Lecar model

In quantitative form the equations are given by:

$$\frac{dV}{dt} = I + 1.1m(1-V) + 2n(-0.7-V) + 0.5(-0.5-V)$$

$$\frac{dn}{dt} = \varepsilon\alpha(n_\infty - n)$$

$$m = 0.5\left(1 + \tanh\left(\frac{V+0.01}{0.15}\right)\right)$$

$$n_\infty = 0.5\left(1 + \tanh\left(\frac{V+0.12}{0.30}\right)\right)$$

$$\alpha = \cosh\left(\frac{V-0.22}{0.60}\right)$$

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The Ermentrout-Kopell canonical model

(also known as the "theta model")

(Ermentrout and Kopell, 1986)

$$\frac{d\theta}{dt} = 1 - \cos\theta + (1+\cos\theta)I(t)$$

The Izhikevich canonical model

(Izhikevich, 2000)

$$\frac{d\theta}{dt} = 1 - \cos\theta + (1+\cos\theta)\left(k_1 + w\right)$$

$$\frac{dw}{dt} = k_2\left(\theta - k_3\right) - k_4 w$$

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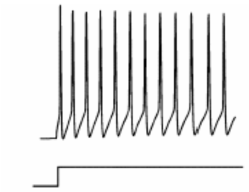
Spiking neuron models

Different spiking neuron models have been proposed. These could be attempts to model the real BN, or as artificial neurons to be used in artificial neural networks.



The previously described real neuron models can generate spikes, and thus are spiking neurons.

Another such model, proposed by Izhikevich (2003) has a structure as shown below:

$$C_m \frac{dV}{dt} = I_m + 0.04V^2 + 5V + 140 - n$$

$$\frac{dn}{dt} = \alpha_n (bV - n)$$

Izhikevich E. (2003). Simple Model of Spiking Neurons. **IEEE Trans. on Neural Networks, 14:6**, 1569-1572.

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Integrate and Fire models

Simple linear model:

Originally proposed by Lapique, back in 1907.

It have been extensively used by Grossberg, Hopfield and many other ANN researchers as it will be shown later.

Basic form of the equation:

$$\frac{dV}{dt} = k_1 (V - k_2) + k_3$$

which is a simple 1D, linear approximation of the previous models.

Non-linear model:

$$\frac{dV}{dt} = f(V - k_2) + k_3$$

Lapicque L. (1907). Recherches quantitatives sur l'excitation e´lectrique des nerfs traite´e comme une polarisation. ***J Physiol Pathol. Gen* 9:** 620–635.

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Mathematical models of BN

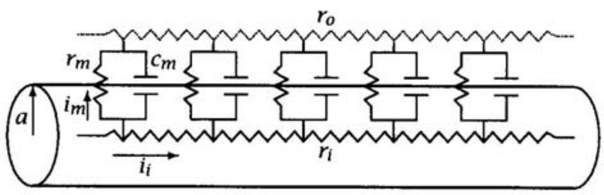
2. Compartmental-based models of BN:

Compartmental-based models are mainly used in order to cope with the difficulties encountered during attempts to model in detail the important parameters and characteristics of real BN.

For a detailed analysis, one has to use partial differential equations

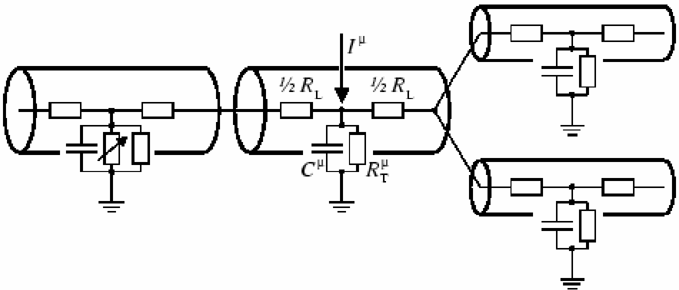
(**cable equation,** $C\frac{\partial V}{\partial t}+I(V,t)=\left(\frac{1}{R}\right)\frac{\partial^2 V}{\partial x^2}$) that can be handled with appropriate numerical mathematics.

A simplified model of a neuronal axon



Compartmental-based models of BN

So, the basic idea is to discretize parts of the neuron, e.g a dendrite or soma, in a compartmental manner, and then apply the required cable equations that can be solved through numerical techniques.



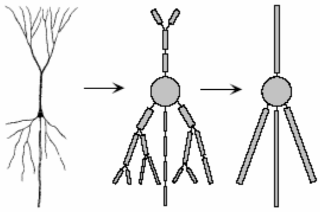
Compartmental-based models of BN

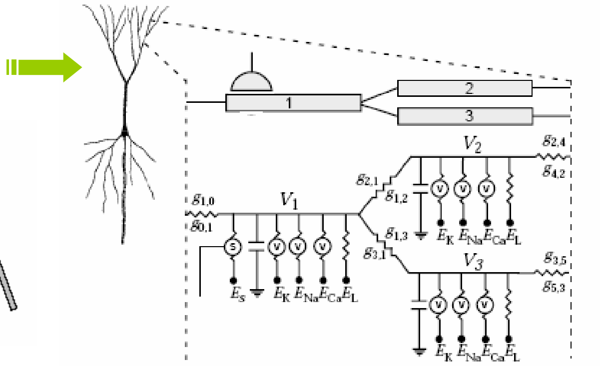
Such models **can accommodate spatial variations** in the membrane response.

They can be done in **different degrees of detailing**.

Typically, they use the linear Integrate and Fire model (which is an oversimplified cable equation) as a basic compartment building block.

$$\frac{dV}{dt} = k_1(V - k_2) + k_3$$





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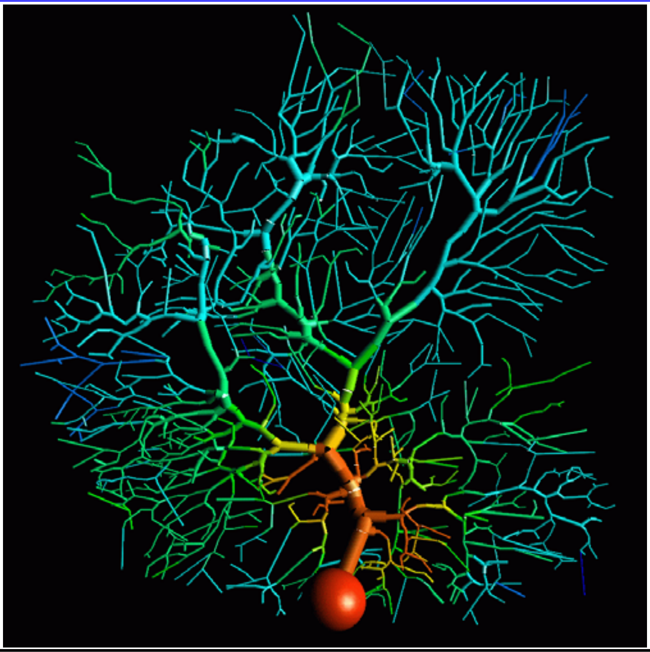
Compartmental-based models of BN



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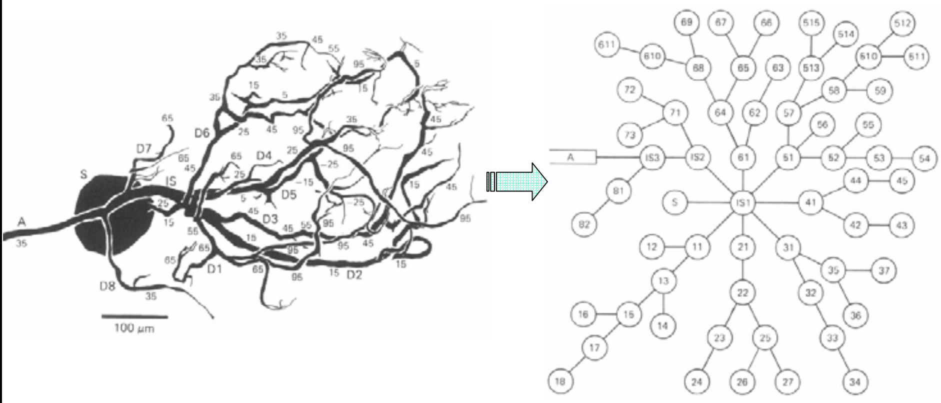
Compartmental-based models of BN



Compartmental-based models of BN

Compartmental-based neuron models and simulations are usually done with suitable computer programs such as:

NEURON
http://www.neuron.yale.edu/neuron/papers/nc97/nctoc.htm
http://www.neuron.yale.edu/neuron/install/install.html

GENESIS (GEneral NEural SImulation System)
http://www.genesis-sim.org/GENESIS/

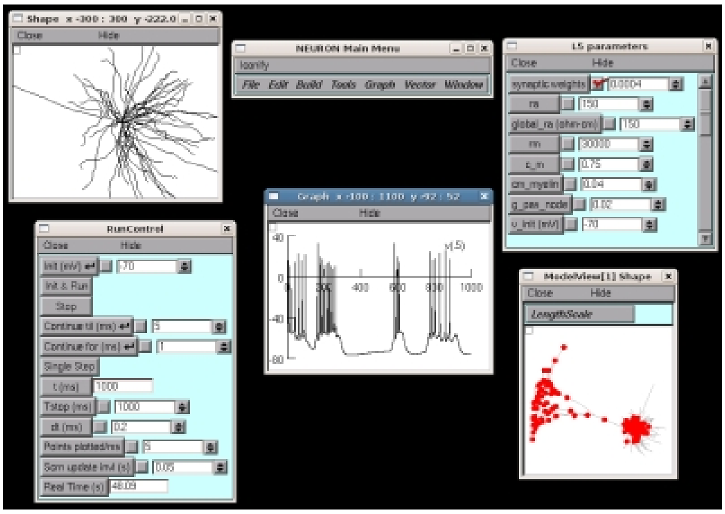
XPPAUT
http://www.math.pitt.edu/~bard/xpp/xpp.html

NODUS
http://www.tnb.ua.ac.be/SOFT/NODUS_info.shtml

Neural Simulation Language
http://www.neuralsimulationlanguage.org/

Compartmental-based models of BN

NEURON SIMULATOR



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Artificial neurons (AN)

These are **simpler** models of BN that are designed to help in building useful ANNs.

From the previous description of BN, we can identify some important and specific characteristics that are common to most neurons and can be used in building AN.

These are:

They are **time-dependent** processors.
They **accumulate** different signals.
They have **multiple inputs – one output** (MISO).
The output is a **train** of approximately **constant amplitude spikes**.
They are largely **non-linear** processors.
They have **many local feedbacks**.
They exhibit **adaptivity**.

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Artificial neurons

They can be implemented in suitable mathematical expressions, in software or in hardware.

In hardware, they can be electrical-electronic, micromechanical, nanomechanical, chemical, optical, ...

Functionally, they are similar to the **cellular automata**.

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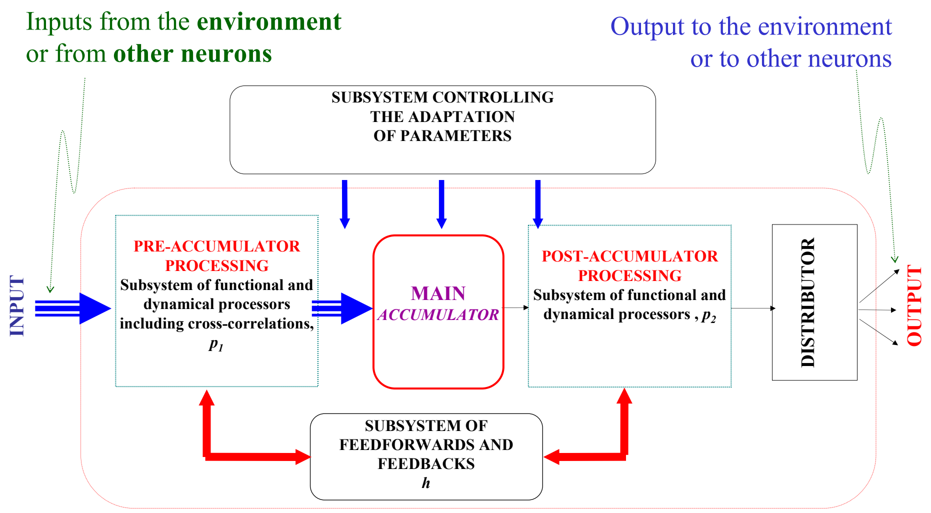
A generic form of a single artificial neuron model



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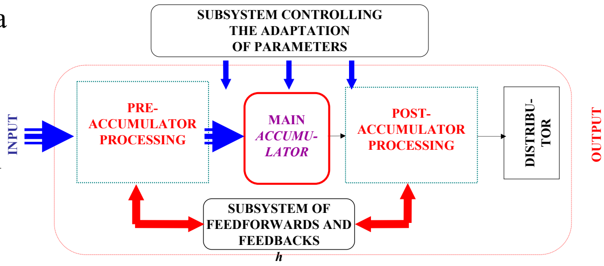
The main usefulness of the previous model is to help a researcher who is interested in exploring the computational properties of artificial neurons and in organizing the field.



It is not an attempt to model in detail the real biological neurons, even though in its general structure resembles the real neuron.

This scheme may be viewed as **a parent model** of most of the existing models, as it will be demonstrated in the following models.

The component blocks may be of considerable complexity, possibly involving **local or remote feedbacks** and **feedforwards** (within the neuron).

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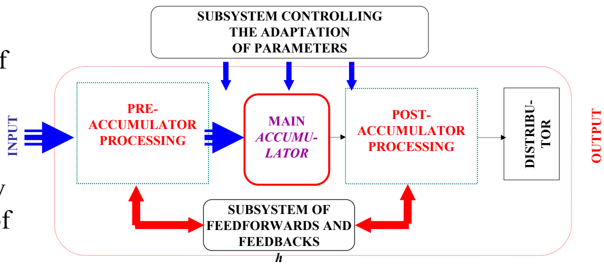
These component blocks may define a subsystem in terms of functionals, dynamical operators (differentiators or integrators) – and hence having internal delays, or may be defined as a combination of functionals, dynamical operators, and algorithmic procedures.



The structural components of the model are identified to be:

- The **pre-main-accumulator** processing subsystem
- The **main-accumulator**
- The **post-main-accumulator** processing subsystem
- The **output distribution subsystem** (distributor)
- The **subsystem of feedback(s)** and **feedforward(s)**
- The **parameter adaptation subsystem**

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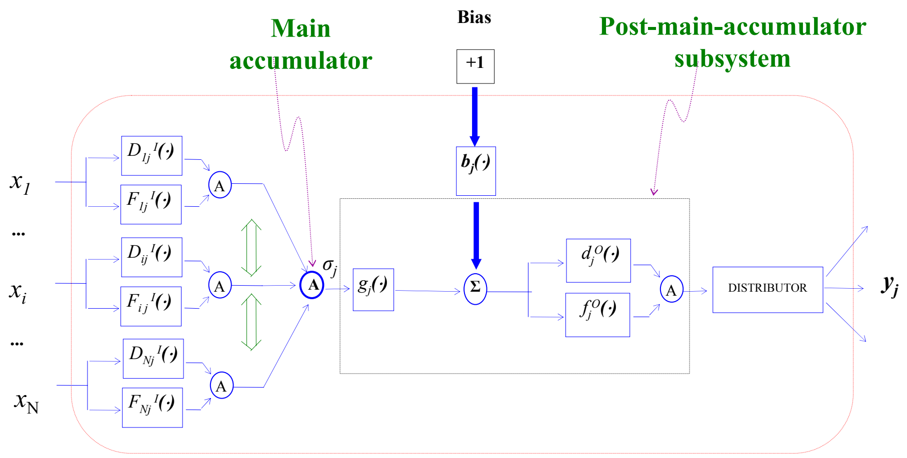
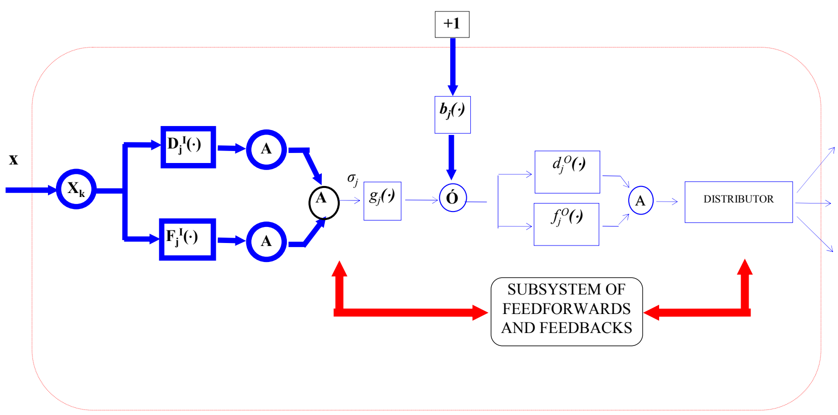
A more detailed form of the generic single-neuron model as used in many ANNs



where,
D's are dynamical transfer operators (involving differentials).
F's are functional operators (involving differentials)



A comment on the distributor

The existing models do not accommodate the possibility for joint regulation and control of the synaptic signal, since each neuron operates locally, basing its action on the local information available.

The artificial neurons however could impose a **conditional processing** or some **joint preprocessing**, governed by the knowledge of the state of one on the other(s).

Such a system becomes more complicated but it may open the way for **new computational paradigms**.

The control of this distributor could even be exerted by an external agent, operating as an overall supervisor.

Such a prospect of course may deviate from the subsymbolic formality and hence weaken the autonomy of the unit. It could however be controlled by some other subsystem of the overall neural network.

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A comment on the distributor

The important issue being that any new scheme will be accepted if it results in more efficient, novel and useful neurocomputational processing.

To make this point clearer, two examples of possible schemes are presented here.

Suppose neuron j sends signals to neurons α, β, γ, δ.
Example rule 1:
Neuron α accepts a signal from neuron j if the ratio of the activation of β to γ is greater than some suitable function of the activation of δ.

Example rule 2:
Neuron α accepts a signal from neuron j if the activation of β and γ is greater than the activation of δ.

Rules like these, (simpler or more complicated) could be used in order to help explore new neurocomputing paradigms.

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Continuous-time mathematical description

$$\frac{d\boldsymbol{u}}{dt} = f(\boldsymbol{u}(t), \boldsymbol{w}(t), \boldsymbol{x}(t)) \quad \text{and} \quad y(t) = f(\boldsymbol{u}(t))$$

where,

t = Time	$t \in \boldsymbol{R}$
$\boldsymbol{u}(t)$ = Internal potential	$\boldsymbol{u}(t) \in \boldsymbol{R}^m$
$\boldsymbol{w}(t)$ = Synaptic weights	$\boldsymbol{w}(t) \in \boldsymbol{R}^q$
$\boldsymbol{x}(t)$ = Output state	$\boldsymbol{x}(t) \in \boldsymbol{R}^n$
$y(t)$ = Neuron output	$y(t) \in \boldsymbol{R}$
$\boldsymbol{\varphi}(.)$ = Internal transfer functions	$\boldsymbol{\varphi}(t) \in \boldsymbol{R}^m$
$f(.)$ = Activation function	$f(.) \in \boldsymbol{R}$

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Discrete-time mathematical description

$$u[\kappa+1] = f(\boldsymbol{u}[\kappa], \boldsymbol{w}[\kappa], \boldsymbol{x}[\kappa]) \quad \text{and} \quad y[\kappa] = f(\boldsymbol{u}[\kappa])$$

where, κ = discrete time counter

and all the other symbols are defined as per previous slide.

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A taxonomy of SNM

BASED ON THE NEURON STRUCTURE

Pre-main-accumulator characteristics

Input signal correlations: 1^{st}, 2^{nd}, or higher order

Input signal dynamics: Linear-nonlinear, 1^{st}, 2^{nd}, or higher order

Main-accumulator characteristics

Sigma, Pi, Sigma-Pi

Radial basis function types

Post-main-accumulator characteristics

Presence of dynamics: Linear-nonlinear, 1^{st}, 2^{nd}, or higher order

Presence and type of functionals:
Linear, ramp, threshold, hard-limiter,
sigmoid, Gaussian, polynomial …

Type of feedback

Single or multiple

Local or remote

Coupling many neurons into a single complicated unit

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A taxonomy of SNM

BASED ON THE DEGREE OF STOCHASTICITY USED IN THE MODEL

Purely deterministic models

Purely stochastic models

Mixed models

BASED ON THE DEGREE OF RESEMBLANCE TO A REAL NEURON

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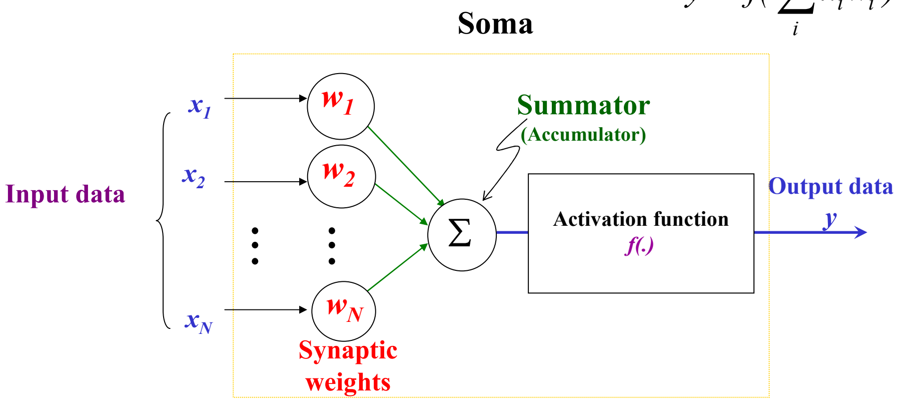
Simple model of an artificial neuron

$$y = f(\sum_{i}^{N} x_i w_i)$$



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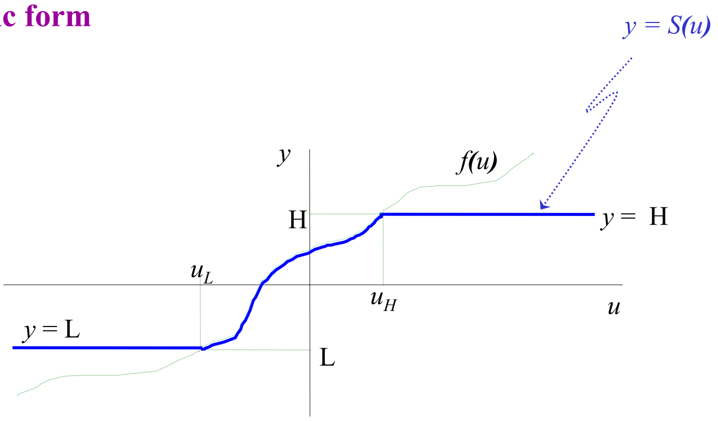
Activation functions

(ΣΥΝΑΡΤΗΣΕΙΣ ΔΡΑΣΤΗΡΙΟΠΟΙΗΣΗΣ)

Generic form



$$y = S(u) = \mathbf{min}(\mathrm{H}, \mathbf{max}(\mathrm{L}, f(u))) = \mathbf{max}(\mathrm{L}, \mathbf{min}(\mathrm{H}, f(u)))$$

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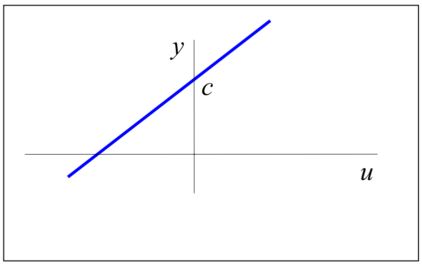
Activation functions

Linear
(Γραμμική)

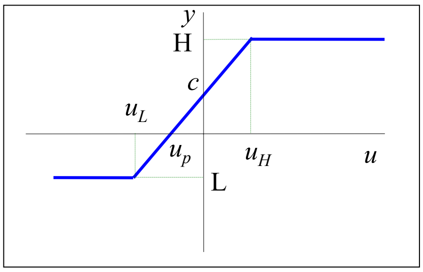
$y = su + c$

where, s = Slope



Ramp
(Σκαλωτή)

$y = \mathbf{min}(\ \mathrm{H}, \mathbf{max}(\ \mathrm{L}, su + c))$



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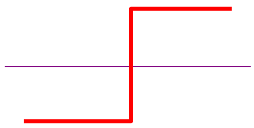
Quantizer, signum, hard limiter, Heavyside
(περιοριστής, ή κβαντιστής, ή ψαλιδιστής)

$$y = \mathbf{sgn}(u) = \begin{cases} +1 & \text{if } \ u \geq 0 \\ -1 & \text{if } \ u < 0 \end{cases}$$



Threshold
(στοιχείο κατωφλίου)

$$y = f(u) = \begin{cases} +1 & \text{if } \ u \geq 0 \\ 0 & \text{if } \ u < 0 \end{cases}$$

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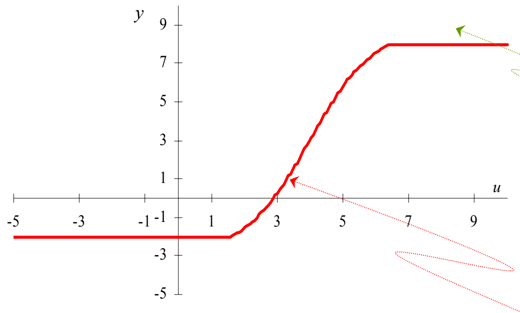
Logistic sigmoid
(Λογιστική-σιγμοειδής)



$$y = c + \frac{A}{1 + e^{-s(u-\gamma)}}$$

$$y = \mathbf{min}(8, \mathbf{max}(-2, -3 + \frac{12}{1 + e^{-(u-4)}}))$$

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Other important activation functions that have been used in AN:

Sinusoidal

$$y = c + A\mathbf{sin}(su)$$

Tanh

$$y = c + A\mathbf{tanh}(su) = c + \frac{A(1 - \mathbf{e}^{-2su})}{(1 + \mathbf{e}^{-2su})}$$

Gabor

$$y = c + Ae^{-s(u-\gamma_1)^{n_1}} \mathbf{cos}(u - \gamma_2)^{n_2}$$

Polynomial Ratio

$$y = c + \frac{Au^n}{1 + u^n}$$

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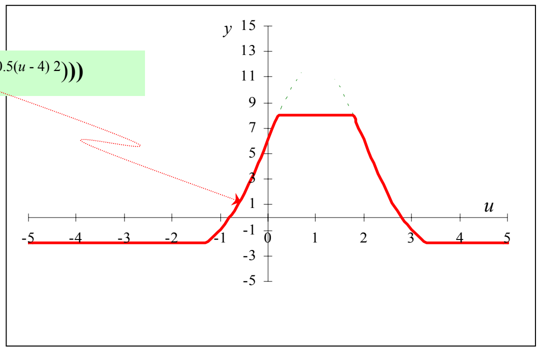
Gaussian $y = \mathrm{c} + \mathrm{A}e^{-s(u-\gamma)^{n}}$

$y = \mathbf{min}(\,8,\ \mathbf{max}(\,-2,\ -3 + 15\boldsymbol{e}^{-0.5(u-4)^2}))$



and many others

The most common models of artificial single neurons

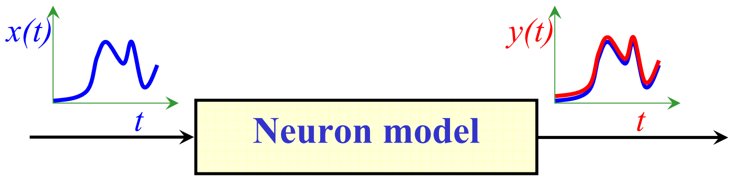
In the following presentation, **the most common single neuron models** (SNM) as used in ANNs will be presented.

The objective is to identify and present the **most representative** and **most influential** in a rather chronological manner.

Obviously, this is not an exhaustive list, but it certainly helps in identifying the important features and enables an interested researcher to proceed to a rigorous comparative simulation if the need arises.

The models will be given in a mathematical description (in an indicial and/or matrix formalism). Many of the models will also be presented as block diagrams.

STATIC (Functional) MODELS



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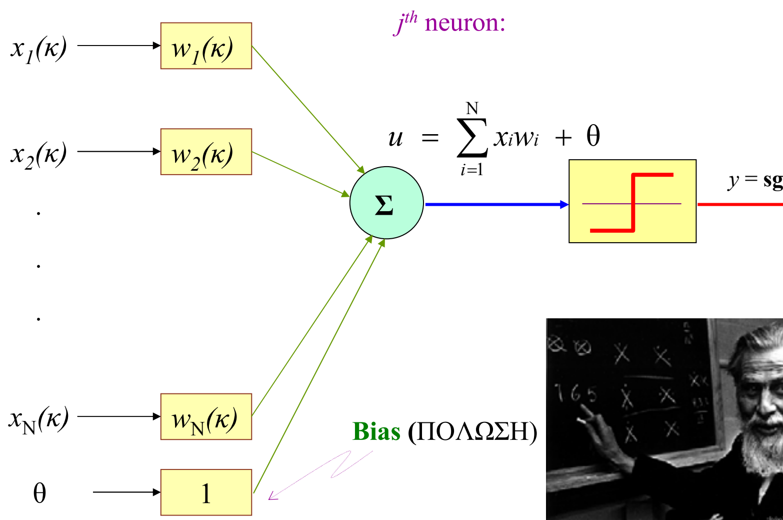
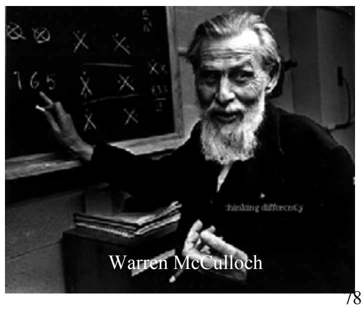
The **McCulloch and Pitts** model (1947)





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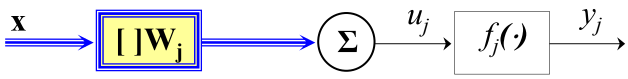
The **McCulloch and Pitts** model (1947)

In a more condensed form:



In indicial form:

$$y_j = f_j(u_j) = f_j\left(\sum_{i=1}^{N} w_{ij}x_i\right) = \mathbf{sgn}\left(\sum_{i=1}^{N} w_{ij}x_i\right)$$

In matrix form:

$$y_j = f_j(u_j) = f_j(\mathbf{w}_j^{\mathrm{T}}\mathbf{x}) = \mathbf{sgn}(\mathbf{w}_j^{\mathrm{T}}\mathbf{x})$$

It is noted that this model is **not dynamic** and thus there is **no time-dependent property growth**.

These types of neuron models are called **static** (or **functional**) neurons as opposed to **dynamic** neurons.

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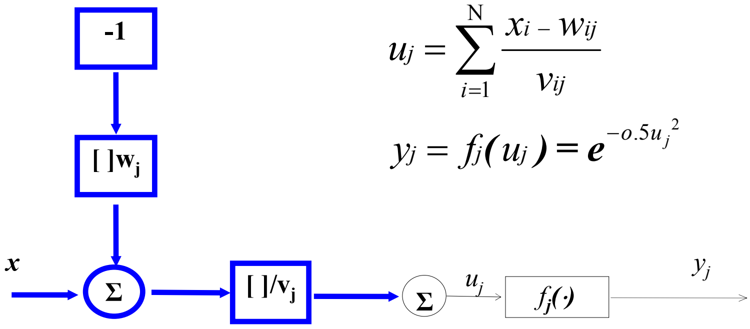
Basic "means - variance connections" model
(Robinson, 1988)



$$u_j = \sum_{i=1}^{N} \frac{x_i - w_{ij}}{v_{ij}}$$

$$y_j = f_j(u_j) = e^{-0.5u_j^2}$$

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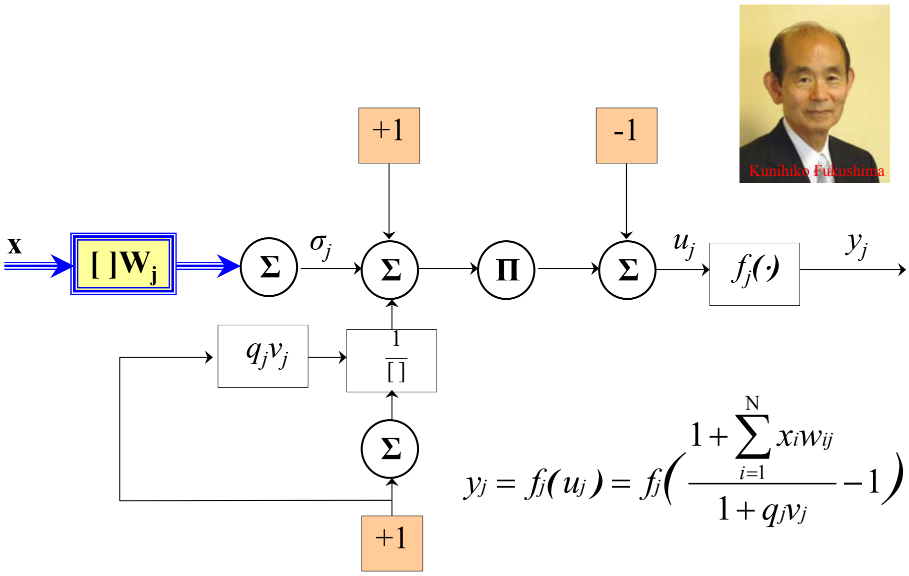
Fukushima model (1988)



$$y_j = f_j(u_j) = f_j\left(\frac{1+\sum_{i=1}^{N} x_i w_{ij}}{1+q_j v_j} - 1\right)$$

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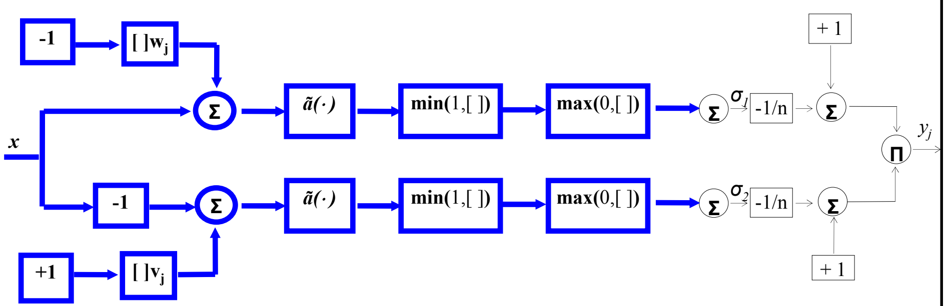
“Product of complements” fuzzy model
(Simpson, 1991)



$$y_j = \left(1 - \frac{1}{N}\sum_{i=1}^{N} \min\left(1, \max\left(0, \gamma\left(x_i - w_{ij}\right)\right)\right)\right)\left(1 - \frac{1}{N}\sum_{i=1}^{N} \min\left(1, \max\left(0, \gamma\left(x_i - v_{ij}\right)\right)\right)\right)$$

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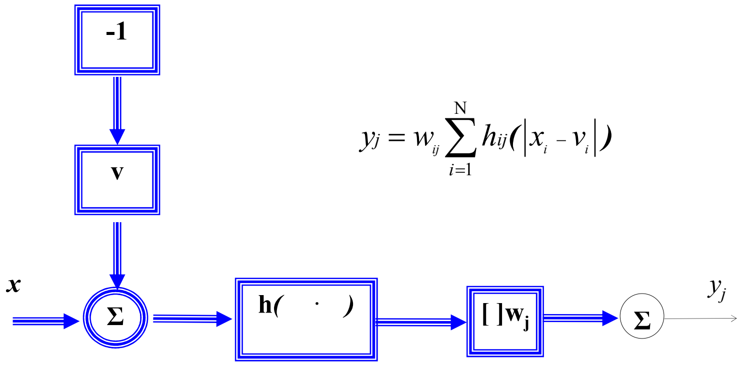
General form of radial basis function neuron model
(Lowe, 1995)



$$y_j = w_{ij}\sum_{i=1}^{N} h_{ij}(|x_i - v_i|)$$

In most of the cases, the form of the function *h(.)* is Gaussian.

$$h(|x_i, v_i|) = e^{\frac{(x_i-v_i)^2}{r^2}}$$

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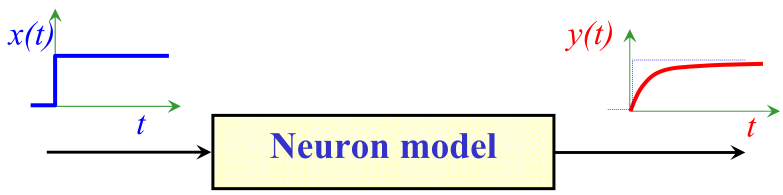
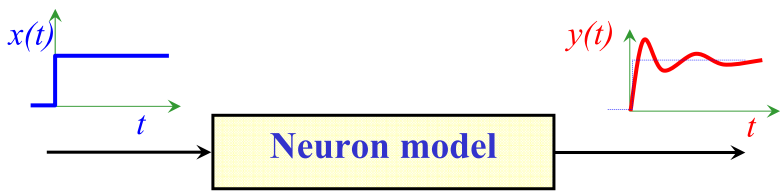
DYNAMICAL MODELS





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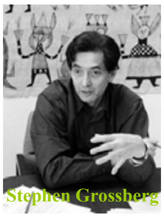
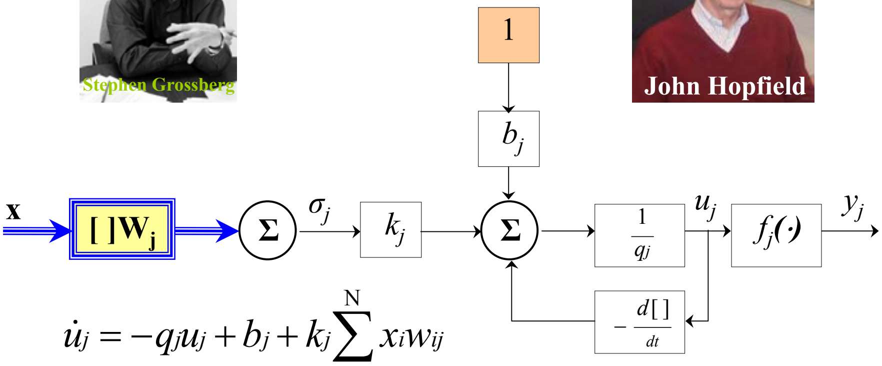
The Leaky Integrator and Fire (LIF) model

Caianiello (1961), (Grossberg, 1968), Amari (1972), (Hopfield, 1982)





$$\dot{u}_j = -q_j u_j + b_j + k_j \sum_{i=1}^{N} x_i w_{ij}$$

$$y_j = f_j(u_j)$$

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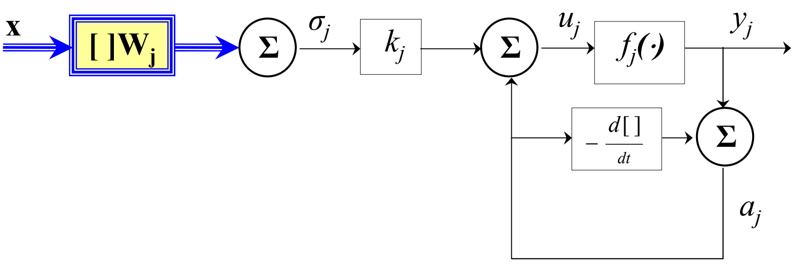
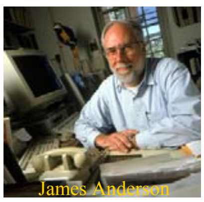
The "**Brain State in a Box**" model

(Anderson et al., 1977)



$$\dot{a}_j = -a_j + f_j\left(a_j + k_j \sum_{i=1}^{N} x_i w_{ij}\right) = -a_j + y_j$$



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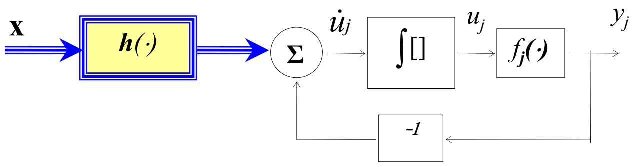
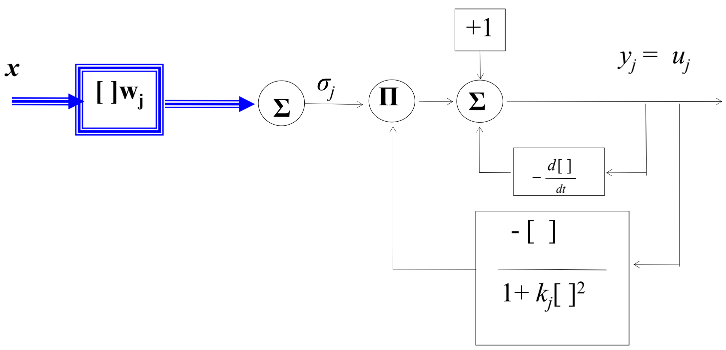
The **Kohonen** (1983) generalized model

$$\dot{u}_j = \sum_{i=1}^{N} h_{ij}(x_i) - f_j(u_j)$$

$$y_j = f_j(u_j)$$



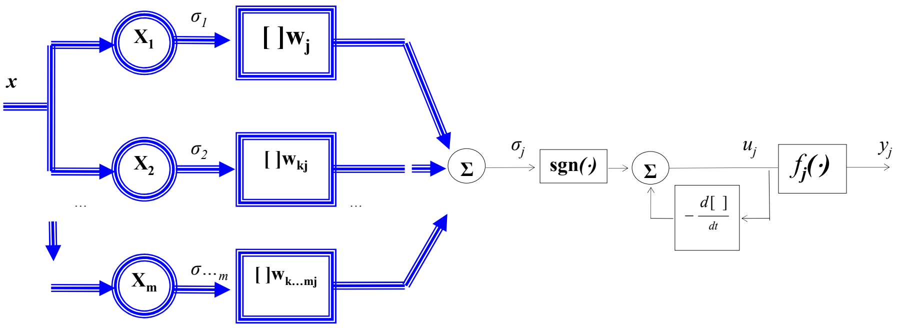
The "Hysteretic neuron" model
(Hoffman and Benson, 1986)



$$\dot{u}_j = 1 - u_j - \left(\frac{u_j}{1+k_j u_j^2}\right)\left(\sum_{i=1}^{N} w_{ij} x_i\right) - f_j(u_j)$$

$$y_j = u_j$$

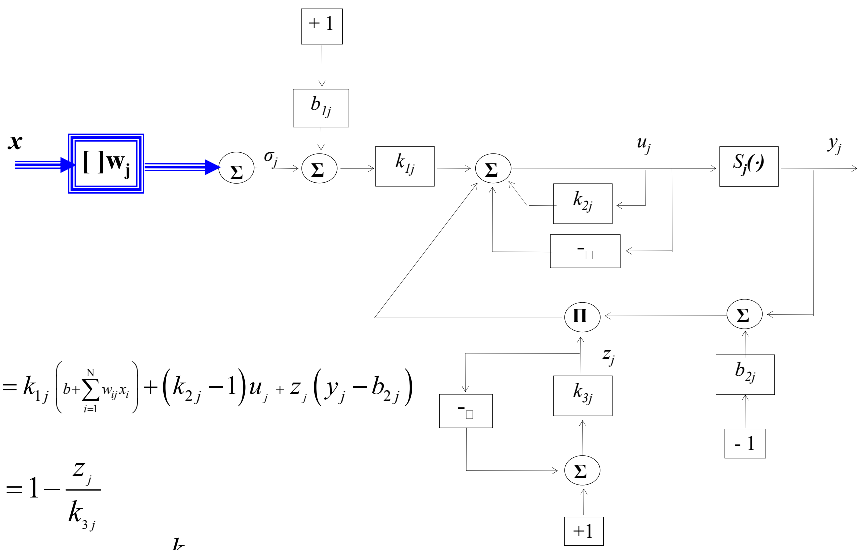
A "Recursive high-order neuron" model
(Kosmatopoulos, 1992)



$$\dot{u}_j = -u_j + \mathbf{sgn}\Big(\sum_{i=1}^{N} x_i w_{ij} + \sum_{k=1}^{N}\sum_{i=1}^{N} x_i x_k w_{ikj} + \ldots + \sum_{m=1}^{N}\ldots\sum_{i=1}^{N} x_i \ldots x_m w_{i\ldots mj}\Big)$$

$$y_j = f_j(u_j)$$

"Transient chaotic neuron" model
(Chen & Aihara, 1995)



$$\dot{u}_j = k_{1j}\left(b + \sum_{i=1}^{N} w_{ij} x_i\right) + \left(k_{2j} - 1\right) u_j + z_j \left(y_j - b_{2j}\right)$$

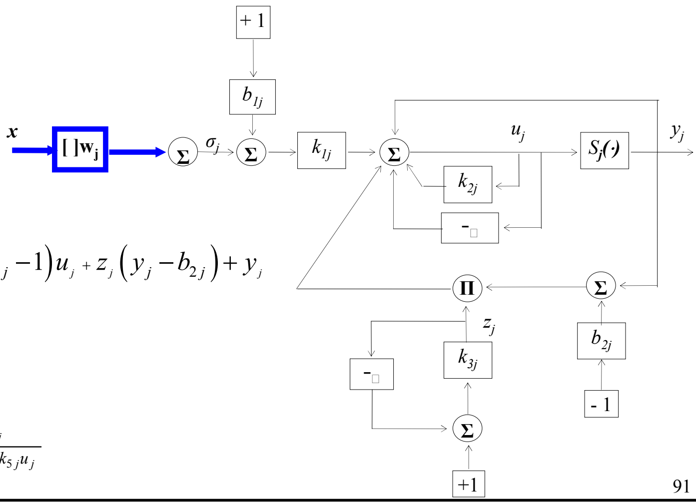
$$\dot{z}_j = 1 - \frac{z_j}{k_{3j}}$$

$$y_j = f_j(u_j) = \frac{k_{4j}}{1 + e^{-k_{5j} u_j}}$$

New combined neuron models

These can be obtained by combining the best features of the previous models.

As an example here is a combination of the Chen and Aihara "transient chaotic neuron" with the "brain state in a box" model (Neocleous & Schizas, 1995).



$$\dot{u}_j = k_{1j}\left(b_{1j}+\sum_{i=1}^{N} w_{ij}x_i\right)+\left(k_{2j}-1\right)u_j + z_j\left(y_j-b_{2j}\right)+y_j$$

$$\dot{z}_j = 1-\frac{z_j}{k_{3j}}$$

$$y_j = f_j(u_j) = \frac{k_{4j}}{1+e^{-k_{5j}u_j}}$$

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Comparison of biological and artificial neurons and networks

BIOLOGICAL NEURONS AND NETWORKS	ARTIFICIAL NEURONS AND NETWORKS
Dense connections **~ (10^{12} neurons)(10^4 synapses) =** **= 10^{16} connections**	**Few connections**
Single neurons are different to one another **Modular structures** **Autonomous local interaction**	**Mostly similar to one another** **Partly modular** **Non-autonomous** **Usually supervision is needed**
Parallel processing **Very little energy consumption** **Non-mathematical or algorithmic operation**	**Mostly serial processing** **Much energy consumption** **Mostly mathematical or algorithmic description**